

Viability of Variable Generalised Chaplygin gas - a thermodynamical approach

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Abstract

The viability of the variable generalised Chaplygin gas (VGCG) model is analysed from the standpoint of its thermodynamical stability criteria with the help of an equation of state, $P = -\frac{B}{\rho^\alpha}$, where $B = B_0 V^{-\frac{n}{3}}$. Here B_0 is assumed to be a positive universal constant, n is a constant parameter and V is the volume of the cosmic fluid. We get the interesting result that if the well-known stability conditions of a fluid is adhered to, the values of n are constrained to be negative definite to make $(\frac{\partial P}{\partial V})_S < 0$ & $(\frac{\partial P}{\partial V})_T < 0$ throughout the evolution. Moreover the positivity of thermal capacity at constant volume c_V as also the validity of the third law of thermodynamics are ensured in this case. For the particular case $n = 0$ the effective equation of state reduces to Λ CDM model in the late stage of the universe while for $n < 0$ it mimics a phantom-like cosmology which is in broad agreement with the present SNe Ia constraints like VGCG model. The thermal equation of state is discussed and the EoS parameter is found to be an explicit function of temperature only. Further for large volume the thermal equation of state parameter is identical with the caloric equation of state parameter when $T \rightarrow 0$. It may also be mentioned that like Santos et al our model does not admit of any critical points. We also observe that although the earlier model of Lu explains many of the current observational findings of different probes it fails to explain the crucial tests of thermodynamical stability.

KEYWORDS : cosmology; chaplygin gas; thermodynamics

1. Introduction

The discovery of cosmic acceleration of the universe [1–3] has added a new challenge for fundamental theories of physics and cosmology. NASA's observations [4,5] show that the kind of matter of which stars and galaxies are made forms less than 5% of the universe's total mass. Several independent observations indicate that about 73% of the total energy density of the universe is in the form of a mysterious dark energy or gravitationally repulsive energy, and about 22% is in the form of non-baryonic cold dark matter particles which clump gravitationally, but which have never been directly detected.

The late acceleration of the universe is often attributed to the presence of one of the most weird and mysterious stuffs termed *dark energy* in the cosmic fluid. But understanding the nature of this dark energy is definitely one of the most challenging

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theoretical problems facing present-day cosmology and people embark on looking out for different plausible alternatives with missionary zeal. For sparing the readers going through the repetition of those arguments and its counterparts (for excellent reviews in this field one is referred to for instance [6, 7]) we skip the details and mention that different variants of Chaplygin gas are also serious contenders for the same. Incidentally many workers in this field strongly feel that it is not enough to keep on generalizing the original Chaplygin gas by introducing new parameters to the theory without much physical basis and then manipulate the arbitrary parameters at will to explain the observational results coming from different cosmic probes but one should also concentrate on basic physics involved. Returning to the multiplicity of arbitrary constants for greater maneuverability one should also check that the different forms of the Chaplygin gas should behave as a closed thermodynamical system. In this context, among many other things the question of its stability is of utmost importance and we are primarily motivated by the consideration if the well known stability criteria may put some stringent conditions on the values of the parameters of the system. In the process we find that some of the claims made by past workers are clearly ruled out when posited against the stability criteria. With this in mind we have in the past worked on the stability problem in some of our works [8,9] and this one also deals with this specific problem in the case of variable modified Chaplygin gas. A Chaplygin type of gas cosmology [10,11] is one of the plausible explanations of recent phenomena, which is a new matter field to simulate dark energy. This type of equation of state(EoS) is not applicable in the case of primordial universe [11,12]. Such equation of state leads to a component which behaves as dust at early stage and as cosmological constant (Λ) at later stage. The form of the equation of state (EoS) of matter is the following,

$$P = -\frac{B}{\rho^\alpha}. \quad (1)$$

Here P corresponds to the pressure of the fluid and ρ is the energy density of that fluid and B is a constant. Recently a variable generalised Chaplygin gas (VGCG) model is proposed and constrained using Union supernovae sample and Barion acoustic oscillation (BAO) [13] assuming the B to depend on scale factor of FRW metric. Now we have taken the above relation as $B = B_0 V^{-\frac{n}{3}}$ where n is an arbitrary constant and B_0 an absolute constant. V is the volume of the fluid. For $n = 0$ the VGCG equation of state reduces to the generalised Chaplygin gas equation of state [11]. In the above mentioned work of Lu they got $n = 0.75$ and $\alpha = 1.53$. Later Lu et al [14] studied the same case with $\alpha = 1$ (VCG) with the help of supernovae union and BAO data. The best fit values of model parameters are found to be $n = 1.30^{+0.46}_{-0.07}(1\sigma) \ ^{+0.74}_{-0.81}(2\sigma)$. Interestingly if SNe Ia Union data is only considered, they obtained $n = 0.13^{+1.42}_{-1.94}(1\sigma) \ ^{+2.14}_{-3.55}(2\sigma)$. Again Sethi *et al* [15] showed that for $\alpha = 1$ the best fit values of n lie in the interval $(-1.3, 2.6)$ [*WMAP 1st Peak + SNe Ia*(3σ)] and $(-0.2, 2.8)$ [*WMAP 3rd Peak + SNe Ia*(3σ)]. So from what has been discussed above we see that the value of n may be both positive or negative. Positive value of n gives a quiescence type of evolution and big rip is avoided while the negative value of n favors a phantom-like Chaplygin gas model which allows for the possibility of the dark energy density increasing with time. Relevant to mention that recently there are some indications that a strongly negative equation of state,

$w \leq -1$, may give a better fit [16–18] with observations.

Literature abounds with work where different forms of Chaplygin gas were taken as dark energy model and tried to fit the different findings of the cosmological probes. But it is not enough to be able to match the observational data. But the thermodynamical viability of the fluid should also be seriously explored. In this context stability of the fluid is crucially important. Previously Santos et al [19] have studied and showed that the generalised Chaplygin gas (GCG) model is thermodynamically stable. The present work is motivated by the consideration if the variable generalised Chaplygin gas (VGCG) is also thermodynamically viable. In the process we find the thermodynamical stability criteria are satisfied subject to the condition that n should be negative definite which apparently contradicts Lu's [13] conclusion for VGCG model. But $\alpha = 1$, the negative value of n lies within the range obtained by Sethi et al [15] and also agreed with another work of Lu et al [14] where they have considered SN Ia Union data only.

Following usual thermodynamical procedure we have derived expressions of different thermal quantities as functions of temperature and volume. We also find that the third law of thermodynamics is satisfied in the case of VGCG. In the cosmological context, we have also found that VGCG is also amenable to a unified picture of dark matter and energy which cools down as the universe expands. Finally, the paper ends with a brief discussion.

2. Energy equation

One may take

$$\rho = \frac{U}{V}, \quad (2)$$

where U and V are the internal energy and volume filled by the fluid respectively.

Now, we try to find out the energy U and pressure P of Variable Generalised Chaplygin gas (VGCG) as a function of its entropy S and volume V . From thermodynamics, one has the following relationship

$$\left(\frac{\partial U}{\partial V} \right)_S = -P. \quad (3)$$

With the help of equations (1), (2) and (3)

$$\left(\frac{\partial U}{\partial V} \right)_S = B_0 V^{-\frac{n}{3}} \frac{V^\alpha}{U^\alpha}, \quad (4)$$

integrating,

$$U = \left[\frac{3B_0(1+\alpha)V^{\frac{3(1+\alpha)-n}{3}}}{3(1+\alpha)-n} + c \right]^{\frac{1}{1+\alpha}}. \quad (5)$$

where the parameter c is the integration function, which may either be a function of entropy S only or a universal constant. The equation (5) may be recast as

$$U = \left[\frac{B_0(1+\alpha)V^{-\frac{n}{3}}}{N} \right]^{\frac{1}{1+\alpha}} V \left[1 + \left(\frac{\epsilon}{V} \right)^N \right]^{\frac{1}{1+\alpha}}, \quad (6)$$

where $N = \frac{3(1+\alpha)-n}{3} > 1$ for $(1+\alpha) > \frac{n}{3}$ for real U and

$$\epsilon = \left[\frac{3(1+\alpha)-n}{3B_0(1+\alpha)} c \right]^{\frac{1}{N}} = \left[\frac{Nc}{B_0(1+\alpha)} \right]^{\frac{1}{N}}, \quad (7)$$

which has dimension of volume. Now the energy density ρ of the VGCG comes out to be

$$\rho = \left[\frac{B_0(1+\alpha)V^{-\frac{n}{3}}}{N} \right]^{\frac{1}{1+\alpha}} \left[1 + \left(\frac{\epsilon}{V} \right)^N \right]^{\frac{1}{1+\alpha}}. \quad (8)$$

In what follows we shall try to obtain the expressions of relevant physical quantities and investigate their behaviour. It is to be mentioned that we get exactly similar expression for density from energy conservation equation with the scale factor given by $a^3(t) = V$.

3. Thermodynamical Behaviour

Using equations (1) and (8) the pressure P of the VGCG may also be determined as a function of entropy S and volume V in the following form

$$P = - \left(B_0 V^{-\frac{n}{3}} \right)^{\frac{1}{1+\alpha}} \left[\frac{N}{(1+\alpha) \left\{ 1 + \left(\frac{\epsilon}{V} \right)^N \right\}} \right]^{\frac{\alpha}{1+\alpha}}. \quad (9)$$

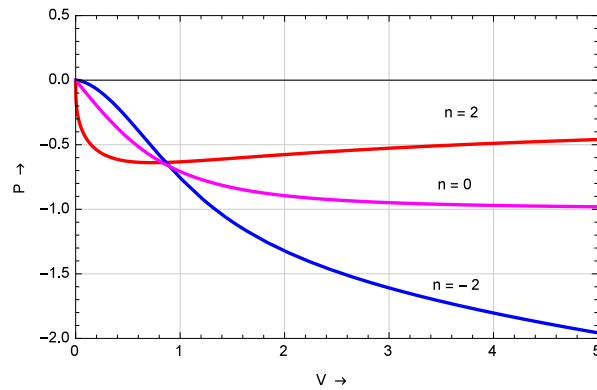


Figure 1: The variation of p and V for different values of n with $B_0 = 1$, $\alpha = 1$ & $c = 1$.

The equation (9) gives a very general expression of pressure where we see that P is always negative. For $n = 0$ the equation (9) becomes

$$P = -\frac{(B_0)^{\frac{1}{1+\alpha}}}{\left\{1 + \left(\frac{c}{B_0 V}\right)^{1+\alpha}\right\}^{\frac{\alpha}{1+\alpha}}}, \quad (10)$$

which reduces to an earlier work of Santos et al [19] for the generalised Chaplygin gas (GCG) model. Again for $\alpha = 1$ in equation (9), we get $P = -\left[\frac{NB_0 V^{-\frac{n}{3}}}{2\left\{1 + \left(\frac{\epsilon}{V}\right)^N\right\}}\right]^{\frac{1}{2}}$ which is identical with our previous work [8] when we consider the Variable Chaplygin gas (VCG) model.

Now using expressions (8) and (9) we get the *effective* equation of state parameter

$$\mathcal{W} = \frac{P}{\rho} = \left\{-1 + \frac{n}{3(1+\alpha)}\right\} \frac{1}{1 + \left(\frac{\epsilon}{V}\right)^N}. \quad (11)$$

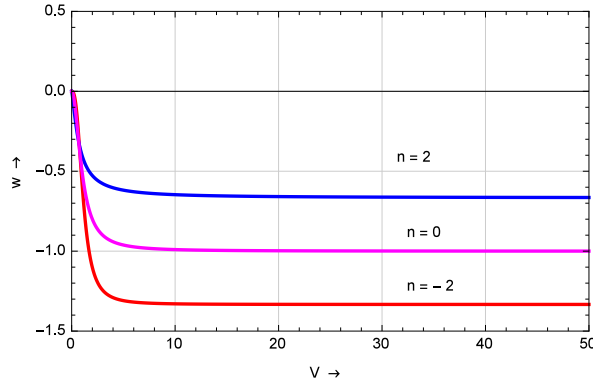


Figure 2: $w \sim V$ graph for different values of n with $B_0 = 1$, $\alpha = 1$ & $c = 1$.

Due to its complexity, we can not arrive at definite conclusions from equation (11), which forces us to look forward to its extreme cases: For small volume, $V \ll \epsilon$, the equation (11) gives

$$P \sim 0. \quad (12)$$

Interestingly we see that for this dust dominated universe, the n has essentially no influence on the pressure P . Again for large volume, $V \gg \epsilon$, the equation (11) becomes

$$\mathcal{W} \approx -1 + \frac{n}{3(1+\alpha)}. \quad (13)$$

since $n < 3(1+\alpha)$. Three possibilities exist for \mathcal{W} depending on the signature of n as (i) $n > 0$, $\mathcal{W} > -1$, which points to a quiescence type of evolution and big rip is avoided in this case, (ii) $n = 0$, $\mathcal{W} = -1$. It represents Λ CDM and (iii) $n < 0$, $\mathcal{W} < -1$ gives a phantom like universe. This is compatible with recent observational

results [16–18]. Influence of n is prominent in this case.

Now we calculate the deceleration parameter of the VGCG fluid from equation (11) as

$$q = \frac{1}{2} + \frac{3}{2} \frac{P}{\rho} = \frac{1}{2} - \frac{3}{2} \left(\frac{N}{1 + \alpha} \right) \frac{1}{1 + \left(\frac{\epsilon}{V} \right)^N}, \quad (14)$$

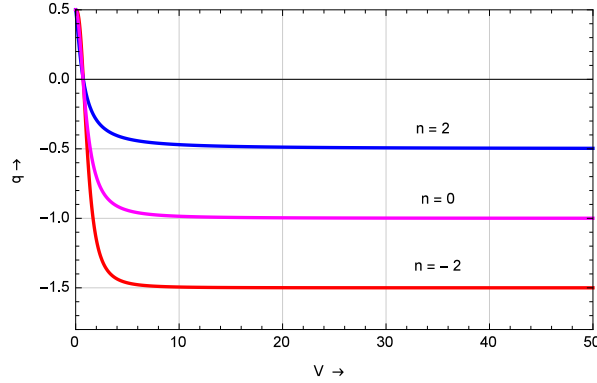


Figure 3: The variation of q and V for different values of n with $B_0 = 1$, $\alpha = 1$ & $c = 1$.

As the equation (14) is very involved in nature we again look for extremal cases as before. For $V \ll \epsilon$, we get

$$q \sim \frac{1}{2}, \quad (15)$$

i.e., universe decelerates for small V . Alternatively for $V \gg \epsilon$, the equation (14) reduces to

$$q \approx -1 + \frac{n}{2(1 + \alpha)}. \quad (16)$$

To sum up when volume is very small there is no influence of n on q , here $q > 0$, universe decelerates. But for large volume, $q < 0$ and this depends on the value of n also. For *flip* in deceleration parameter the flip volume (V_f) becomes

$$V_f = \epsilon \left[\frac{(1 + \alpha)}{2(1 + \alpha) - n} \right]^{\frac{1}{N}}. \quad (17)$$

The above equation dictates that $n < 2(1 + \alpha)$, which interestingly does not violate our previous restriction on n . From equation (17) it follows that for V_f to have real value $n < 2(1 + \alpha)$, otherwise *flip* does not occur. This also follows from the fig - 3 where only $n < 4$ allows *flip* (for $\alpha = 1$). Alternatively the inequality $n < 2(1 + \alpha)$ gives the condition of acceleration. Thus for decelerates $V < V_f$ and for acceleration $V > V_f$.

If v_s be the velocity of sound equation (9) gives

$$v_s^2 = \left(\frac{\partial P}{\partial \rho} \right)_s = \frac{N\alpha}{(1+\alpha) \left\{ 1 + \left(\frac{\epsilon}{V} \right)^N \right\}} - \frac{nN}{n + (3N+n) \left(\frac{\epsilon}{V} \right)^N}. \quad (18)$$

The equation (18) offers some interesting possibilities: this gives $v_s^2 = 0$ at dust dominated universe and for large volume we get

$$v_s^2 = -1 + \frac{n}{3(1+\alpha)}. \quad (19)$$

We shall presently see that from the thermodynamical stability conditions n becomes negative, leading to a phantom type of universe [20]. Moreover the equation (19) gives an imaginary speed of sound for $\alpha > 0$, leading to a perturbative cosmology. One need not be too sceptic about it because it favours structure formation [21].

It is tempting to make some comparison with a recent work of Myong [22] where holographic dark energy, Chaplygin gas and tachyon model with constant potential are briefly discussed *vis-a-vis* their implications as regards squared acoustic speeds. This is all the more relevant because signature of the squared speeds is a key factor in determining the stability criteria. It is observed that the squared speed of tachyonic field and chaplygin gas are always positive while if the condition of future event horizon is assumed *a priori* as the IR cut off but for holographic dark energy it is always negative definite.

4. Stability Criteria

At this stage one may check the thermodynamic stability conditions of a fluid during its evolution. For this it is necessary to find if (i) the pressure reduces both for an adiabatic and isothermal expansion [23] $\left(\frac{\partial P}{\partial V} \right)_S < 0$ & $\left(\frac{\partial P}{\partial V} \right)_T < 0$ and (ii) as also to examine if the thermal capacity at constant volume is positive.

Equations (1) and (9) give

$$\left(\frac{\partial P}{\partial V} \right)_S = \frac{P}{3V(1+\alpha)} \left[\frac{3N\alpha \left(\frac{\epsilon}{V} \right)^N}{1 + \left(\frac{\epsilon}{V} \right)^N} - n \right]. \quad (20)$$

Since P is always negative, $\left(\frac{\partial P}{\partial V} \right)_S < 0$ for $n < \frac{3N\alpha \left(\frac{\epsilon}{V} \right)^N}{1 + \left(\frac{\epsilon}{V} \right)^N}$, but this is not possible at the late stage of evolution because RHS of inequality is a function of volume. It will be a better option to get the negative value of $\left(\frac{\partial P}{\partial V} \right)_S$ throughout the evolution for both $\alpha > 0$ and $n < 0$ simultaneously.

Now we have discussed some special cases as follows :

(i) A cursory glance at the equation (20) shows that n and α can not be at once zero because that makes $\left(\frac{\partial P}{\partial V} \right)_S = 0$ leading to a severe restriction on the stability of the fluid. In this case the pressure becomes constant through any adiabatic change of volume. Moreover B_0 here behaves like a cosmological constant and consequently we get a de-Sitter type of metric for late universe. When $\alpha = 0$ and $n \neq 0$, $\left(\frac{\partial P}{\partial V} \right)_S = \frac{n}{3} B_0 V^{-(1+\frac{n}{3})}$, i.e., for $n < 0$, $\left(\frac{\partial P}{\partial V} \right)_S < 0$. Evidently, $n \leq 0$ implies that the

pressure becomes more and more negative with volume. This agrees well with the observational results [16].

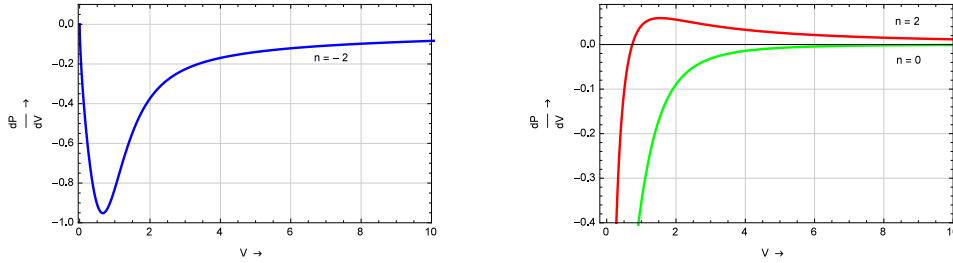
(ii) For $n = 0$ and $\alpha \neq 0$, the equation (20) becomes identical to an earlier work of Santos et al [19]. In this case

$$\left(\frac{\partial P}{\partial V}\right)_S = \alpha \frac{P}{V} \left(\frac{\epsilon}{V}\right)^{1+\alpha} \left\{1 + \left(\frac{\epsilon}{V}\right)^{1+\alpha}\right\}^{-1}, \quad (21)$$

which gives $\left(\frac{\partial P}{\partial V}\right)_S < 0$ for $\alpha > 0$. Again for $\alpha = 1$ and $n \neq 0$, equation (20) reduces to our previous work [8] where

$$\left(\frac{\partial P}{\partial V}\right)_S = \frac{P}{6V} \left[(6-n) \left\{ 1 - \frac{1}{1 + \left(\frac{\epsilon}{V}\right)^N} \right\} - n \right] \quad (22)$$

This is the case of Variable Chaplygin gas(VCG).



(a) This graph clearly show that $\left(\frac{\partial P}{\partial V}\right)_S < 0$ for $n < 0$ throughout the evolution.

(b) Here we have seen that for $n > 0$, VGCG does not stable throughout the evolution.

Figure 4: The variations of $\left(\frac{\partial P}{\partial V}\right)_S$ and V are shown

It is clear from equation (20) that for $\alpha > 0$, n should be negative for $\left(\frac{\partial P}{\partial V}\right)_S < 0$ throughout the evolution. It may be pointed out that this conclusion accords with the work of Sethi et al [15] where they have obtained the best fit values of n well lying between $(-1.3, 2.6)$ [WMAP 1st peak + SN Ia (3σ)] for $\alpha = 1$. But there is another work of Lu [13] where the Union SNe Ia data and Sloan Digital Sky Survey(SDSS) baryon acoustic peak to constrain the Variable Generalised Chaplygin Gas (VGCG) model is studied and the best fit values of $n = 0.75$ for $\alpha = 1.53$ are obtained. From what has been discussed above we see that Lu's conclusion about the positivity of n is inadmissible when confronted with thermodynamical stability criteria. It also follows from fig-4.

Now we have to examine if $\left(\frac{\partial P}{\partial V}\right)_T \leq 0$ as well. We will show in the next section that for $n < 0$ this condition may also be satisfied.

Thermal EoS:

One should also verify the positivity of thermal capacity at constant volume c_V where $c_V = T \left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{\partial U}{\partial T}\right)_V = V \left(\frac{\partial p}{\partial T}\right)_V$. Next the temperature T of the Variable

modified Chaplygin gas as a function of volume V and entropy S may be determined from the relation $T = \left(\frac{\partial U}{\partial S}\right)_V$. As we have considered B_0 to be an absolute constant, the equation (5) gives

$$T = \frac{V^{1-N-\frac{n}{3(1+\alpha)}}}{1+\alpha} \left[\frac{B_0(1+\alpha)}{N} + V^{-N}c \right]^{-\frac{\alpha}{1+\alpha}} \left(\frac{\partial c}{\partial S} \right)_V. \quad (23)$$

We have previously seen that c may be a either universal constant or function of entropy S . If we consider c to be an absolute constant T becomes zero for all values of volume and pressure and it makes the stability criteria questionable. Naturally we are forced to take $c \equiv c(S)$ and temperature varies with expansion. We have no *a priori* knowledge of the functional dependence of c . From physical considerations, however, we know that this function must be such as to give positive temperature and cooling along an adiabatic expansion and so we choose that $\left(\frac{\partial c}{\partial S}\right) > 0$.

In the literature [24] Santos et al have considered Jacobian identity to calculate the expression of c for the case of Modified Chaplygin gas, where they assumed that $\left(\frac{\partial P}{\partial V}\right)_T = 0$. But in our approach we have considered the dimensional analysis to derive the expression of c because later we will show in this process that $\left(\frac{\partial P}{\partial V}\right)_T < 0$ for $n < 0$. This is more general approach in our sense.

Now from dimensional analysis, we observe from equation (5) that

$$[U] = [c]^{\frac{1}{1+\alpha}}. \quad (24)$$

Using $[U] = [T][S]$, we can write

$$[c] = [T]^{1+\alpha} [S]^{1+\alpha} \quad (25)$$

It is difficult to get an analytic solution of c from equation (25). So as a trial case, we take an empirical expression of c which is a function of entropy only such that

$$c = (\tau)^{1+\alpha} S^{1+\alpha}, \quad (26)$$

where τ is a constant having the dimension of temperature only. Now

$$\frac{dc}{dS} = (1+\alpha) (\tau)^{1+\alpha} S^\alpha. \quad (27)$$

Using equation (23) and (27), we get the expression of temperature

$$T = V^{1-N-\frac{n}{3(1+\alpha)}} \left[\frac{B_0(1+\alpha)}{N} + V^{-N}c \right]^{-\frac{\alpha}{1+\alpha}} (\tau)^{1+\alpha} S^\alpha \quad (28a)$$

$$= \tau \left\{ 1 - \frac{1}{1 + \left(\frac{\epsilon}{V}\right)^N} \right\}^{\frac{\alpha}{1+\alpha}}. \quad (28b)$$

and from equation (28), the entropy is

$$S = \frac{\left[\frac{B_0(1+\alpha)}{N} V^N \right]^{\frac{1}{1+\alpha}} \left(\frac{T}{\tau^{1+\alpha}} \right)^{\frac{1}{\alpha}}}{\left\{ 1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right\}^{\frac{1}{1+\alpha}}}, \quad (29)$$

It follows from equation (29) that for positive and finite entropy one should have $0 < T < \tau$, *i.e.*, τ represents the maximum temperature.

Moreover for $T = 0$, $S = 0$ showing that the third law of thermodynamics is satisfied in this case.

Now using equation (29) we get the expression of thermal heat capacity as

$$c_V = T \left(\frac{\partial S}{\partial T} \right)_V = \frac{\left[\frac{B_0(1+\alpha)}{N} V^{-\frac{n}{3}} \right]^{\frac{1}{1+\alpha}} \left(\frac{T}{\tau^{1+\alpha}} \right)^{\frac{1}{\alpha}}}{\alpha \left\{ 1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right\}^{\frac{2+\alpha}{1+\alpha}}} . \quad (30)$$

Since $0 < T < \tau$ and $\alpha > 0$, $c_V > 0$ is always satisfied irrespective of the value of n . This also ensures the positivity of α . It is interesting to note that when the temperature goes to zero c_V vanishes in agreement with the third law of thermodynamics.

If we now put $\alpha = 1$, *i.e.* the equation (30) reduces to the expression of c_V of our previous work [8]. Again for $n = 0$, we get the identical expression of c_V found by Santos et al [19].

To end the section a final remark may be in order. While *positivity* of specific heat is strongly desirable *vis-a-vis* when dealing with special relativity, in a recent communication Luongo *et al* [25] argued that in a FRW type of model like the one we are discussing a negative specific heat at constant volume and a vanishingly small specific heat at constant pressure (c_P) are also compatible with observational data. In fact they have derived the most general cosmological model which is agreeable with the $c_V < 0$ and $c_P \sim 0$ values obtained for the specific heats of the universe and showed, in addition, that it also overcomes the fine-tuning and the coincidence problems of the Λ CDM model.

We now proceed to find an expression for internal energy of the VGCG as a function of V and T with the help of equations (5), (26) and (29) as

$$U = V \left\{ \frac{\frac{B_0(1+\alpha)}{N} V^{-\frac{n}{3}}}{1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}}} \right\}^{\frac{1}{1+\alpha}} . \quad (31)$$

Now using (1), (2) and (31) the Pressure becomes

$$P = - \left(B_0 V^{-\frac{n}{3}} \right)^{\frac{1}{1+\alpha}} \left(\frac{N}{1+\alpha} \right)^{\frac{\alpha}{1+\alpha}} \left\{ 1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right\}^{\frac{\alpha}{1+\alpha}} . \quad (32)$$

The equation (32) shows that for $\alpha = 1$, the expression for pressure is identical to our previous work [8] as expected. Further we see that for $n = 0$ the above expression goes to an earlier work of Santos et al [19]. With the help of equations (31) and (32) we get the thermal EoS parameter of VGCG as

$$\omega = - \frac{N}{1+\alpha} \left\{ 1 - \left(\frac{T}{\tau} \right)^{\frac{1+\alpha}{\alpha}} \right\} . \quad (33)$$

Here thermal EoS parameter is a function of temperature only. In the early era of the universe when temperature is very high, *i.e.*, at $T \rightarrow \tau$, the equation (33) gives

that $\omega \sim 0$, *i.e.*, $P \sim 0$ representing a dust dominated universe. This is identical with the equation (12). On the other hand, at the late stage of the universe, at very low temperature, *i.e.*, $T \rightarrow 0$, the equation (33) simplifies to $\omega \approx -1 + \frac{n}{3(1+\alpha)}$. This is similar to equation (13). Thus thermodynamical state represented by equations (12) and (13) are essentially same at both dust and the late stage of the universe.

Now from equation (32) we shall examine if $(\frac{\partial P}{\partial V})_T \leq 0$. Thus we get

$$\left(\frac{\partial P}{\partial V}\right)_T = \frac{nB_0V^{-(1+\frac{n}{3})}}{3(1+\alpha)} \left\{ \frac{B_0(1+\alpha)}{N} V^{-\frac{n}{3}} \right\}^{\frac{1}{1+\alpha}} \left\{ 1 - \left(\frac{T}{\tau}\right)^{\frac{1+\alpha}{\alpha}} \right\}^{\frac{\alpha}{1+\alpha}}. \quad (34)$$

which clearly shows that the value of n should be negative for $(\frac{\partial P}{\partial V})_T < 0$ throughout the evolution. It is very interesting to note that for $n = 0$, $(\frac{\partial P}{\partial V})_T = 0$. At this stage we digress to make a comparison with the works of Santos et al [19, 24] referred to earlier. In their cases for GCG model they got pressure as a function of temperature only while for MCG they assumed the same, *i.e.*, in both the cases $(\frac{\partial P}{\partial V})_T = 0$. But in our case for $n < 0$, $(\frac{\partial P}{\partial V})_T < 0$ implying that the isobaric curves of our VGCG model do not coincide with its isotherms in the diagram of thermodynamic states. This is definitely a significant improvement in our analysis.

Thus we conclude that both $(\frac{\partial P}{\partial V})_S$ and $(\frac{\partial P}{\partial V})_T$ are negative for $n < 0$ which is in accordance with the stability condition of thermodynamics. One can very briefly check if any critical points exist in our model. We find that as in the case of Santos our model also does not posses any critical points.

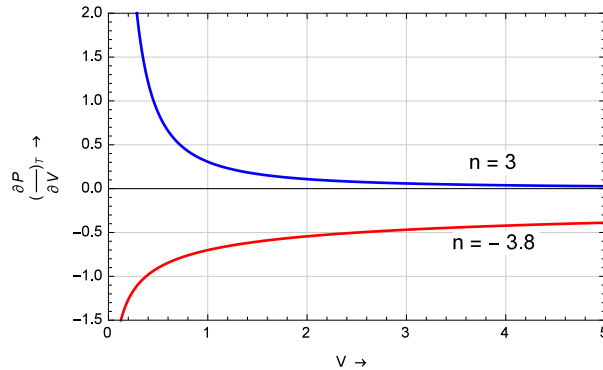


Figure 5: The variation of $(\frac{\partial P}{\partial V})_T$ and V for different values of n . We have considered here $B_0 = 1$, $\alpha = 1$ & $c = 1$.

Now from equation (1) we get

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{\alpha B_0 V^{-\frac{n}{3}}}{\rho^{\alpha+1}} \frac{\partial \rho}{\partial T}. \quad (35)$$

and from equation (31) we can write

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{N\rho}{1+\alpha}. \quad (36)$$

Now using equations (35) and (36) into the wellknown relation of thermodynamics [23]

$$\left(\frac{\partial U}{\partial V}\right)_T = T \left(\frac{\partial P}{\partial T}\right)_V - P, \quad (37)$$

we get

$$\frac{\partial T}{T} = \frac{B_0 \alpha (1 + \alpha)}{N V^{\frac{n}{3}} \rho^{1+\alpha} - B_0} \frac{\partial \rho}{\rho}. \quad (38)$$

Integrating,

$$\beta T = \left\{ 1 - \frac{B_0 (1 + \alpha)}{N V^{\frac{n}{3}} \rho^{1+\alpha}} \right\}^{\frac{\alpha}{1+\alpha}}, \quad (39)$$

where β is an integration constant and hence we get the expression of energy density as

$$\rho = \left\{ \frac{\frac{B_0 (1 + \alpha)}{N} V^{-\frac{n}{3}}}{1 - (\beta T)^{\frac{1+\alpha}{\alpha}}} \right\}^{\frac{1}{1+\alpha}}. \quad (40)$$

The same result can be obtained from equation (31) if we identify $\beta = \frac{1}{\tau}$. It is interesting to point out that the empirical expression of c which was taken in equation (26) from the consideration of dimensional analysis is justified.

As we are more concerned with the accelerating universe which is a late stage phenomena where the results are similar to our previous work relating to VMCG model, *i.e.*, at the late stage VMCG apparently reduces to VCG model.

We can also express the maximum temperature τ as a function of the initial conditions of the expansion. If we consider that the initial conditions at $V = V_0$ are $\rho = \rho_0$, $P = P_0$ and $T = T_0$, then we can get from (5)

$$c = \left\{ \rho_0^{1+\alpha} - \frac{B_0 (1 + \alpha)}{N} V_0^{-\frac{n}{3}} \right\} V_0^{1+\alpha}. \quad (41)$$

With the help of equations (6), (7) and (41), we obtain the energy density ρ and the pressure P as a function of the volume V as

$$\rho = V^{-\frac{n}{3(1+\alpha)}} \rho_0 \left[\frac{B_0 (1 + \alpha)}{N \rho_0^{1+\alpha}} + \left\{ 1 - \frac{B_0 (1 + \alpha)}{N \rho_0^{1+\alpha}} V^{-\frac{n}{3}} \right\} \left(\frac{V_0}{V} \right)^{1+\alpha} V^{\frac{n}{3}} \right]^{\frac{1}{1+\alpha}}, \quad (42)$$

and

$$P = - \frac{B_0^{\frac{1}{1+\alpha}} \left(\frac{B_0}{\rho_0^{1+\alpha}} \right)^{\frac{\alpha}{1+\alpha}} V^{\frac{n}{3(1+\alpha)}}}{\left[\frac{B_0 (1 + \alpha)}{N \rho_0^{1+\alpha}} + \left\{ 1 - \frac{B_0 (1 + \alpha)}{N \rho_0^{1+\alpha}} V^{-\frac{n}{3}} \right\} \left(\frac{V_0}{V} \right)^{1+\alpha} V^{\frac{n}{3}} \right]^{\frac{\alpha}{1+\alpha}}}. \quad (43)$$

Now equations (32), (42) and (43) can be written as function of the reduced parameters ε , v , p , κ and t such that

$$\begin{aligned}\varepsilon &= \frac{\rho}{\rho_0}, & v &= \frac{V}{V_0}, & p &= \frac{P}{B_0^{\frac{1}{1+\alpha}}} \\ \kappa &= \frac{B_0(1+\alpha)}{N\rho_0^{1+\alpha}}, & t &= \frac{T}{T_0}, & \tau^* &= \frac{\tau}{T_0}\end{aligned}\quad (44)$$

The equations (32), (42) and (43) can now be expressed in the reduced units respectively as

$$p = -\left(\frac{N}{1+\alpha}\right)^{\frac{\alpha}{1+\alpha}} V^{-\frac{n}{3(1+\alpha)}} \left\{1 - \left(\frac{t}{\tau^*}\right)^{\frac{1+\alpha}{\alpha}}\right\}^{\frac{\alpha}{1+\alpha}}, \quad (45)$$

$$\varepsilon = V^{-\frac{n}{3(1+\alpha)}} \left[\kappa + \left\{1 - \kappa V^{-\frac{n}{3}}\right\} \frac{V^{\frac{n}{3}}}{v^{1+\alpha}} \right]^{\frac{\alpha}{1+\alpha}}, \quad (46)$$

$$p = -\frac{\kappa^{\frac{\alpha}{1+\alpha}} \left(\frac{NV^{-\frac{n}{3}}}{1+\alpha}\right)^{\frac{\alpha}{1+\alpha}}}{\left[\kappa + \left\{1 - \kappa V^{-\frac{n}{3}}\right\} \frac{V^{\frac{n}{3}}}{v^{1+\alpha}}\right]^{\frac{\alpha}{1+\alpha}}}. \quad (47)$$

At $P = P_0$, $V = V_0$ and $T = T_0$, we have $t = 1$ and $v = 1$ and we get from equations (45) and (47),

$$\begin{aligned}p_0 &= -\kappa^{\frac{\alpha}{1+\alpha}} \left(\frac{N}{1+\alpha}\right)^{\frac{\alpha}{1+\alpha}} V_0^{-\frac{n}{3}}, \\ &= -\left(\frac{N}{1+\alpha}\right)^{\frac{\alpha}{1+\alpha}} V_0^{-\frac{n}{3(1+\alpha)}} \left\{1 - \left(\frac{t}{\tau^*}\right)^{\frac{1+\alpha}{\alpha}}\right\}^{\frac{\alpha}{1+\alpha}},\end{aligned}\quad (48)$$

hence κ and τ^* can be determined as follows:

$$\kappa = V_0^{\frac{n}{3}} \left\{1 - \frac{1}{(\tau^*)^{\frac{1+\alpha}{\alpha}}}\right\}, \quad (49)$$

and

$$\tau^* = \frac{1}{\left(1 - \kappa V_0^{-\frac{n}{3}}\right)^{\frac{\alpha}{1+\alpha}}}. \quad (50)$$

Interestingly, we have seen that τ^* depends on both κ , V_0 and n also. For $n = 0$, all the above equations reduce to the equations of Santos et al [19, 24]. At the present epoch, $\kappa = \frac{B_0(1+\alpha)}{N\rho_0^{1+\alpha}}$, therefore, $\rho_0 = \left\{\frac{B_0(1+\alpha)}{N\kappa}\right\}^{\frac{1}{1+\alpha}}$. If we consider that the temperature $\tau = 10^{32}K$ (temperature at the Planck era) and $T_0 = 2.7K$ (the temperature of the present epoch), the ratio, $\tau^* = \frac{\tau}{T_0} = 3.7 \times 10^{31}$. So the ratio κ will be

$$\kappa = V_0^{\frac{n}{3}} \left\{ 1 - \frac{1}{(3.7 \times 10^{31})^{\frac{\alpha}{1+\alpha}}} \right\} \approx V_0^{\frac{n}{3}}, \quad (51)$$

because $\alpha > 1$. Again from (31), for the case of present epoch when temperature T is very small (i.e. $T \rightarrow 0$),

$$\rho_0 \approx \left\{ \frac{B_0(1+\alpha)}{NV^{\frac{n}{3}}} \right\}^{\frac{1}{1+\alpha}} \approx \left\{ \frac{B_0(1+\alpha)}{N\kappa} \right\}^{\frac{1}{1+\alpha}}, \quad (52)$$

The same result can be obtained from equation (8) for large volume. Thus from equation (26), at present epoch, the energy density ρ of the universe filled with VCG must be very close to $\left\{ \frac{B_0(1+\alpha)}{N\kappa} \right\}^{\frac{1}{1+\alpha}}$.

5. Discussion

We have here studied a very general type of exotic fluid, termed ‘Variable Generalised Chaplygin gas and discussed its cosmological implications, mainly its thermodynamical stability. Although the exhaustive analysis of the latest cosmological observations points to the existence of some form of dark energy in the universe it is very difficult to choose among the merits of its different forms at least from the observational results. In fact most of the alternatives meet the energy budget. So for the specific case of different types of Chaplygin gas we have taken recourse to the investigation if the gas in question behaves as a thermodynamically closed system with those values of the parameters to meet the observational demands. In this context we have taken the stability criteria of the gas to check its consistency and have followed the standard prescription : $(\frac{\partial P}{\partial V})_S < 0$, $(\frac{\partial P}{\partial V})_T < 0$ and $c_V > 0$. Interestingly this dictates that the new parameter, n introduced in VGCG should be negative definite. While this conclusion accords with the results obtained by Sethi et al [15] where the best fit value of n lie between $(-1.3, 2.6)$ for $\alpha = 1$ but the constraint obtained by Lu et al [13] is $n > 0$. This finding is untenable when judged from our results as the Lu’s model becomes thermodynamically unstable in that case.

Again our model shows that at the dust dominated universe, the EoS becomes $P = 0$ and at the late stage $\mathcal{W} = -1 + \frac{n}{3(1+\alpha)}$. As pointed earlier, from the thermodynamical stability conditions we find that $n < 0$, which favours a phantom model with its attendant big rip problem. But later the phantom like evolution is found to be compatible with SNe Ia observations and CMB anisotropy measurements [20].

We have also studied the deceleration parameter where we calculate the flip volume and shows the flip is possible for $n < 2(1+\alpha)$. For dust dominated universe, we get $q = \frac{1}{2}$ whereas at late stage $q < 0$ for $n < 0$.

We have derived the expressions of the temperature as well as entropy. It is to be noted that at $T = 0$ both the entropy and thermal capacity of VGCG vanish as in conformity with the third law of thermodynamics. We have studied both the thermal and the caloric EoS which shows that both $0 < T < \tau$.

From equation (34), it is seen that for $n < 0$, $(\frac{\partial P}{\partial V})_T < 0$ throughout the evolution which is also a stability condition. But we see that for $n = 0$, $(\frac{\partial P}{\partial V})_T = 0$. This

is the case for Santos et al where they considered *a priori* $(\frac{\partial P}{\partial V})_T = 0$ to calculate the expression of c . Actually for GCG model they got pressure as a function of temperature only while for MCG they assumed the same, i.e., in both the cases $(\frac{\partial P}{\partial V})_T = 0$. But in our case for $n < 0$ automatically $(\frac{\partial P}{\partial V})_T < 0$ implying that the isobaric curves of our VGCG model do not coincide with its isotherms in the diagram of the thermodynamic states. However in our model critical points are absent in line with the conclusion of the work of Santos et al.

We have also expressed the equation of states in terms of reduced parameters. Again using wellknown thermodynamic relation (37) we get an expression of the energy density of cosmic fluid which exactly tallies with the expression (31).

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